### North Carolina Standard Course of Study Mathematics (1992–1993)

### Subject: Grade 6

#### Goal 1: The learner will demonstrate an understanding and use of rational numbers.

- 1.1: Use models to relate percents to fractions and decimals; record, read and explain.
- 1.2: Use models and pictures to demonstrate ratios, proportions and percents; explain relationships.
- **1.3:** Read, write and use numbers in various forms, including fractions, decimals, percents and exponential notation, choosing the appropriate form for a given task.
- **1.4:** Find the prime factorization of a number less than 100.
- **1.5:** Use prime factorization to investigate common factors and common multiples, using a calculator when appropriate.
- **1.6:** Explore relationships among whole numbers, fractions, decimals and percents, using money, concrete models or a calculator.
- 1.7: Explore other numeration systems, including ancient number systems and alternate bases.
- 1.8: Explore the meaning of integers in real life situations.
- Goal 2: The learner will demonstrate an understanding and use properties and relationships of geometry.
  - 2.1: Build models of 3-dimensional figures (prisms, pyramids, cones and other solids); describe and record their properties.
  - **2.2:** Recognize and use angles (interior, exterior, complementary, supplementary) and pairs of lines, including skew lines.
  - **2.3:** Construct congruent segments and congruent angles. Construct bisectors of line segments, using a straight edge and compass.
  - 2.4: Identify and distinguish among similar, congruent and symmetrical figures; name corresponding parts.
  - 2.5: Recognize the results of translations, reflections and rotations using technology when appropriate.
  - 2.6: Explore changes in a shape through stretching, shrinking and twisting.
  - 2.7: Recognize geometry in the environment (e.g. art, nature, architecture).

# Goal 3: The learner will demonstrate an understanding of patterns, relationships and pre-algebra.

- 3.1: Represent number patterns in a variety of ways, including the use of calculators and computers.
- **3.2:** Use patterns to explore the rules for divisibility.
- **3.3:** Use graphs and tables to represent relations of ordered pairs, using a calculator or a computer where appropriate; describe the relationships.
- **3.4:** Identify and use patterning as a strategy to solve problems.
- **3.5:** Use realistic examples or models to represent concepts and properties of variables, expressions and equations. (Identity property of zero, Identity property of one.)
- **3.6:** Use the order of operations to simplify numerical expressions, verifying the results with a calculator or computer.

#### Goal 4: The learner will demonstrate an understanding and use of measurement.

- **4.1:** Convert measures of length, area, volume, capacity and weight expressed in a given unit to other units in the same measurement system.
- **4.2:** Determine whether a given measurement is precise enough for the specific situation; determine when estimates are sufficient for the measurement situation.
- **4.3:** Explore the relationship of areas of triangles with the same base and height. Use models to demonstrate formulas for finding areas of triangles, parallelograms and circles.
- 4.4: Explore the effect on area and perimeter when changing one or two of the dimensions of a rectangle.
- **4.5:** Use models, develop the concept of volume for rectangular solids as the product of the area of the base and the height.
- 4.6: Estimate solutions and solve problems related to volumes of rectangular solids.

#### Goal 5: The learner will solve problems and reason mathematically.

- 5.1: Use an organized approach to solve non-routine and increasingly complex problems involving numeration, geometry, pre-algebra, measurement, graphing, computation, probability and statistics.
- 5.2: Analyze problem situations and apply appropriate strategies for solving them.
- 5.3: Use inductive and deductive reasoning to solve problems.
- 5.4: Select an appropriate method for solving problems including estimation, observation, formulas, mental math, paper and pencil calculation, calculators and computers.
- 5.5: Make conjectures and arguments and identify various points of view.

## Goal 6: The learner will demonstrate an understanding and use of graphing, probability and statistics.

- 6.1: Create and evaluate graphic representations of data including circle graphs.
- **6.2:** Use measures of central tendency (mean, median and mode) and range to describe meaningful data; compare two sets of unequal data.
- 6.3: Display data using computer software and explore the use of spreadsheets.
- **6.4:** In the coordinate plane, locate ordered pairs in meaningful situations, using whole numbers, fractions and decimals.
- 6.5: Estimate the likelihood of certain events from experiments with graphical data.
- 6.6: Interpret a statistical statement and discuss the extent to which the results of a sample can be generalized.
- 6.7: Find probabilities of simple events and discuss the implications.
- **6.8:** Design an experiment to test a theoretical probability; record and explain results.

#### Goal 7: The learner will compute with rational numbers.

- **7.1:** Use whole number operations to solve real world applications, demonstrating competence with and without calculators (multiplication and division up to 3 digits by 2 digits).
- 7.2: Select appropriate strategies to solve a variety of application problems and justify the selection.
- 7.3: Divide decimal numbers, record results and explain procedures, (1- and 2-digit divisors).
- 7.4: Within a context, estimate results and apply appropriate operations with decimals.
- **7.5:** Use models and pictures to demonstrate multiplication and division of fractions and mixed numbers; record and explain results.
- 7.6: Within a meaningful context, use estimation and operations with fractions less than one.
- 7.7: In problem situations, use estimation and operations with fractions and mixed numbers.
- 7.8: In meaningful contexts develop the concept of adding and subtracting integers; record results.
- 7.9: Translate word problems into number sentences that use integers.
- 7.10: Estimate percents in real world situations and justify the estimate.
- 7.11: Use mental math to solve problems involving simple fractions, decimals and percents.
- **7.12:** Relate common fractions to frequently used percents; estimate and calculate using these percents (multiples of 10, 25,  $33\frac{1}{3}$ ,  $66\frac{2}{3}$ , 75).
- **7.13:** Use ratios and proportions to explore probability and other interesting problems; discuss reasonableness of results.

### Subject: Grade 7

#### Goal 1: The learner will demonstrate an understanding and use of real numbers.

- 1.1: Use models to represent positive and negative rational numbers.
- **1.2:** Compare and order rational numbers in meaningful contexts.
- **1.3:** Express whole numbers in scientific notation; convert scientific notation to standard form; explore the use of scientific notation.
- 1.4: Use exponential notation to express prime factorization of numbers less than 100.
- 1.5: Within meaningful contexts use estimation techniques with rational numbers; justify the strategy chosen.
- **1.6:** Use geometric models to develop the meaning of the square and the positive square root of a number; estimate square roots and find square roots on the calculator.
- 1.7: In meaningful contexts, relate concepts of ratio, proportion and percent.

# Goal 2: The learner will demonstrate an understanding and use properties and relationships of geometry.

- 2.1: Make constructions of perpendicular and parallel lines using a straight edge and compass.
- 2.2: Use the concepts and relationships of geometry to solve problems.
- 2.3: Use models to develop the concept of the Pythagorean Theorem.
- 2.4: Identify applications of geometry in the environment.
- 2.5: Given models of 3-dimensional figures, draw representations.
- 2.6: Given the end, side, and top views of 3-dimensional figures, build models.
- 2.7: On a coordinate plane, graph shapes and congruent figures.

#### Goal 3: The learner will demonstrate an understanding of pre-algebra.

- **3.1:** Describe, extend, analyze and create a wide variety of patterns to investigate relationships and solve problems.
- **3.2:** Use concrete materials as models to develop the concept of operations with variables.
- **3.3:** Use concrete, informal and formal methods to model and solve simple equations.
- **3.4:** Investigate and evaluate algebraic expressions using mental calculations, pencil and paper and calculators where appropriate.
- 3.5: Given a simple equation, formulate a problem; solve and explain.

#### Goal 4: The learner will demonstrate an understanding and use of measurement.

- 4.1: Apply measurement concepts and skills as needed in problem-solving situations.
- 4.2: Make judgments about degree of precision needed and reasonableness of results in measurement situations.
- 4.3: Use models to develop the concept and formula for surface area of rectangular solids and cylinders.
- **4.4:** Use models to develop the concept of volume for prisms/cylinders as the product of area of the base and height.
- **4.5:** Use models to explore the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.
- 4.6: Estimate answers; then solve problems related to volume.

#### Goal 5: The learner will solve problems and reason mathematically.

- 5.1: Use an organized approach and a variety of strategies to solve increasingly complex, non-routine problems.
- **5.2:** Use calculators and computers in problem-solving situations as appropriate.
- 5.3: Discuss alternate strategies, evaluate outcomes and make conjectures and generalizations based on problem situations.
- 5.4: Use concrete or pictorial models involving spatial visualization to solve problems.
- 5.5: Identify and solve problems that require proportional reasoning.
- 5.6: Solve problems involving interpretation of graphs, including inferences and conjectures.

## Goal 6: The learner will demonstrate an understanding and use of graphing, probability and statistics.

- 6.1: Create, compare and evaluate, both orally and in writing, different graphic representations of the same data.
- **6.2**: Construct a box plot (box-and-whisker) by ordering data, identifying the median, quartiles and extremes.
- 6.3: Evaluate appropriate uses of different measures of central tendency.
- 6.4: Draw inferences and construct convincing arguments based on analysis of data.
- 6.5: Investigate and recognize misuses of statistical or numerical information.
- 6.6: Show all possible outcomes by making lists, tree diagrams and frequency distribution tables.
- 6.7: Explain the relationship between experimental results and mathematical expectations.
- **6.8:** Find the probability of simple events using experiments, random number generation, computer simulation and theoretical methods.
- 6.9: Explore permutations and combinations in applications.

#### Goal 7: The learner will compute with real numbers.

- 7.1: Select appropriate operations, strategies and methods of solving a variety of application problems using positive rational numbers; justify the selection.
- **7.2:** Estimate and solve problems using ratio, proportion and percent; select and use appropriate methods; explain the process used.
- **7.3:** Apply concepts of ratio, proportion and percent to real life situations such as consumer applications, science and social studies.
- 7.4: Use real world examples and models to represent multiplication and division of integers; record and explain procedures used.
- **7.5:** Use operations with integers in relevant problem situations.

### Subject: Grade 8

#### Goal 1: The learner will demonstrate an understanding and use of real numbers.

- 1.1: Explore the real number system by describing and using various forms of numbers in realistic situations.
- 1.2: Use appropriate estimation techniques in meaningful situations; justify the technique.
- **1.3:** Use and explain definitions and laws of exponents to write expressions in equivalent forms.
- **1.4:** Use scientific notation to express whole numbers and numbers less than one, using a calculator when appropriate.
- **1.5:** Investigate irrational numbers and their representations on a calculator as they arise from problem situations.
- **1.6:** Describe the properties of terminating, repeating and nonrepeating decimals and be able to convert fractions to decimals and decimals to fractions.
- 1.7: Explore the absolute value of a number using the number line.

### Goal 2: The learner will demonstrate an understanding and use properties and relationships of geometry.

- **2.1:** Use the Pythagorean Theorem to find the missing side of a right triangle; use a calculator when appropriate.
- 2.2: Solve problems related to similar figures, using indirect measures to determine missing data.
- 2.3: Draw 3-dimensional figures from different perspectives (top, side, front).
- 2.4: On a coordinate plane, graph similar figures, reflections and translations.
- 2.5: Explore the triangle congruency relationships: ASA, SSS, SAS.
- 2.6: Explore the relationships of the angles formed when parallel lines are cut by a transversal.
- 2.7: Solve problems that relate geometric concepts to real world situations.

#### Goal 3: The learner will demonstrate an understanding of pre-algebra.

- **3.1:** Describe, extend, analyze and create a wide variety of geometric and numerical patterns, such as Pascal's triangle or the Fibonacci sequence.
- **3.2:** Identify and define the commutative, associative and distributive properties; give examples and explain their meanings.
- **3.3:** Analyze representations of data with tables, graphs, verbal rules and equations to explore properties and relationships.
- **3.4:** Using patterns and algebraic methods, solve problems, including those with integers.
- **3.5:** Generate ordered pairs with and without a calculator and graph the linear equation.
- **3.6:** Informally investigate non-linear equations and inequalities.
- 3.7: Given a formula, make appropriate substitutions and solve for one unknown.

### Goal 4: The learner will demonstrate an understanding and use of measurement.

- **4.1:** Estimate the answer, then solve complex problems that include applications of measurement; determine precision and check for reasonableness of results.
- 4.2: Determine the number of significant digits and the greatest possible error in measurement situations.
- **4.3:** Select an appropriate unit and tool to find a measurement based upon the degree of accuracy required and the nature of the problem situation.
- 4.4: Find the surface area and volume of pyramids, prisms, cylinders and cones, with and without models.
- 4.5: Explore the effect on plane and solid figures when a dimension of the figure is changed.

#### Goal 5: The learner will solve problems and reason mathematically.

- 5.1: Use an organized approach and a variety of strategies to solve increasingly complex non-routine problems.
- 5.2: Use calculators and computers in problem-solving situations as appropriate.
- 5.3: Make and evaluate conjectures and arguments, using deductive and inductive reasoning.
- 5.4: Investigate open-ended problems, formulate questions and extend problem-solving situations.
- 5.5: Represent situations verbally, numerically, graphically, geometrically and symbolically.
- 5.6: Use ratio and proportion to solve problems.

## Goal 6: The learner will demonstrate an understanding and use of graphing, probability and statistics.

- **6.1:** Collect data involving two variables and display on a scatter plot; interpret results.
- **6.2:** Compute the mean, interpret it, explain its sensitivity to extremes and explain its use in comparison with the median.
- **6.3:** Apply knowledge of statistics in problem-solving situations, selecting an appropriate format for presenting data.
- 6.4: Use mathematical probabilities and experimental results to make predictions and decisions.
- **6.5:** Evaluate arguments based on data and investigate reasons why an inference made from a set of data can be invalid (biased or unbiased).
- **6.6:** Find the probability of simple and compound events using experiments, computer simulations, random number generation and theoretical methods.

#### Goal 7: The learner will compute with real numbers.

- 7.1: Select appropriate operations, strategies and methods of solving a variety of application problems using real numbers; justify the selection.
- 7.2: In meaningful contexts, develop the laws of exponents; solve problems involving exponentiation.

### Subject: Pre-Algebra

## Goal 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

- **1.1:** Read, write, and use numbers in various forms, including fractions, decimals, percents, and exponential notation.
- **1.2:** Write numbers in scientific notation, using a calculator when appropriate.
- **1.3:** Use patterns to explore the tests for divisibility.
- 1.4: Use exponential notation to express the prime factorization of a number.
- 1.5: Find the greatest common factor (GCF) and the least common multiple (LCM) of two or more numbers.
- **1.6:** Compare two or more rational numbers.
- 1.7: Estimate sums, differences, products, and quotients of real numbers.
- 1.8: Use a table of squares or a calculator to find the square root of a number.
- **1.9:** Estimate the square root of a number.
- 1.10: Demonstrate an understanding of the relationships between fractions, decimals, percents, and ratios.
- 1.11: Relate common fractions to frequently used percents; estimate and calculate using these percents.
- **1.12:** Find the missing term of a proportion.
- 1.13: Find the percent of a given number.
- 1.14: Find what percent one number is of another.
- 1.15: Find a number when a percent of it is given.
- **1.16:** Write fractions as equivalent, terminating, or repeating decimals.
- **1.17:** Write terminating or repeating decimals as equivalent fractions.
- 1.18: Find the absolute value of a real number.
- 1.19: Use laws of exponents to write expressions in equivalent forms.

# Goal 2: The learner will demonstrate an understanding and use properties and relationships of geometry.

- 2.1: Classify angles (interior, exterior, vertical, complementary, supplementary) and pairs of lines (parallel, perpendicular, skew) and solve problems involving them.
- 2.2: Identify and distinguish among similar, congruent, and symmetric figures; name corresponding parts.
- **2.3:** Find the lengths of the sides of similar figures.
- **2.4:** Use the Pythagorean Theorem to find the missing side of a right triangle and use a table of square roots, calculator or mental math to approximate square roots.
- **2.5:** Find the lengths of the missing sides of  $30^{\circ}-60^{\circ}-90^{\circ}$  and  $45^{\circ}-45^{\circ}-90^{\circ}$  right triangles given the length of one side.
- 2.6: Graph similar geometric figures on a coordinate plane.
- 2.7: Solve problems that relate geometric concepts to real world situations.

#### Goal 3: The learner will demonstrate an understanding of the language of algebra.

- **3.1:** Use order of operations to simplify numerical expressions, verifying the results with a calculator or computer.
- **3.2:** Identify and use the commutative, associative, and distributive properties.
- **3.3:** Evaluate algebraic expressions.
- **3.4:** Solve simple equations algebraically.
- 3.5: Solve simple inequalities and graph their solutions.
- 3.6: Use graphs and tables to represent relations and functions of ordered pairs.
- **3.7:** Describe, extend, analyze and create a variety of geometric and numerical patterns to investigate relationships and solve problems.
- 3.8: Given an equation, formulate a problem.
- **3.9:** Write an equation which could be used to solve a given problem.
- 3.10: Given a rule or function that describes a linear equation, generate ordered pairs and graph the equation.
- 3.11: Given a formula, make appropriate substitutions and solve for one unknown.

#### Goal 4: The learner will demonstrate an understanding and use of measurement.

- 4.1: Apply measurement concepts and skills involving area, perimeter, and circumference in problem-solving situations.
- **4.2:** Determine the effect on area and perimeter when changing one or two of the dimensions of a rectangle.
- 4.3: Find the surface area of rectangular solids, pyramids, cones and cylinders.
- 4.4: Find the volume of prisms, pyramids, cylinders and cones.
- 4.5: Estimate solutions and solve problems related to volumes of rectangular solids.

#### Goal 5: The learner will solve problems and reason mathematically.

- **5.1:** Represent situations verbally, numerically, graphically, geometrically, or symbolically and use a variety of strategies to solve non-routine problems.
- 5.2: Discuss alternate strategies, evaluate outcomes, and make conjectures and generalizations based on problem situations.
- **5.3:** Use concrete or pictoral models to solve problems.
- 5.4: Solve problems involving the interpretation of graphs including inferences and conjectures.
- 5.5: Make and evaluate conjectures and arguments, using deductive and inductive reasoning.
- 5.6: Investigate open-ended problems, formulate questions, and extend problem-solving situations.
- 5.7: Use proportional reasoning to solve problems.

#### Goal 6: The learner will demonstrate an understanding and use of probability and statistics.

- 6.1: Create, compare, and evaluate different graphic representations of the same data.
- **6.2:** Use measures of central tendency (mean, mode, and median) and range to describe meaningful data; compare two sets of unequal data.
- 6.3: Evaluate appropriate uses of different measures of central tendency in problem-solving situations.
- 6.4: Draw inferences and construct convincing arguments based on an analysis of data.
- 6.5: Investigate and recognize misuses of statistical or numerical data.
- **6.6:** Find the mathematical and experimental probability of simple and compound events using experiments, random number generation, computer simulation, and theoretical methods.
- 6.7: Collect data involving two variables and display them on a scatter plot.
- 6.8: Interpret data displayed on a scatter plot.
- 6.9: Use mathematical probability and experimental results for making predictions and decisions.

#### Goal 7: The learner will use fractions, decimals, percents, and integers to solve problems.

- 7.1: Use operations with whole numbers, decimals, fractions and integers to solve problems demonstrating appropriate use of mental math, computation, and calculators.
- **7.2:** Select appropriate operations and strategies for solving a variety of applied problems involving fractions, decimals, and percents and justify the selection, using calculators, mental math, and estimates.
- 7.3: Estimate the answer to and solve problems using ratio, proportion, and percent.
- 7.4: Use ratio, proportion, and percent to solve real-world problems.

### Subject: Algebra I-A

#### Goal 1: The learner will use the language of algebra.

- **1.1:** Simplify numerical expressions.
- **1.2:** Use "order of operations" to simplify numerical expressions.
- **1.3:** Use grouping symbols to indicate order of operations.
- 1.4: Evaluate variable expressions.
- 1.5: Simplify exponential expressions.
- 1.6: Write expressions in exponential form.
- 1.7: Translate word phrases into variable expressions.
- 1.8: Translate variable expressions and equations into word phrases.

#### Goal 2: The learner will identify and apply properties of real numbers.

- 2.1: Identify and apply the properties of zero for both addition and multiplication.
- 2.2: Identify and apply the property of one for multiplication.
- **2.3:** Identify and apply the commutative and associative properties of addition and multiplication to simplify expressions or computational processes.
- 2.4: Identify and apply the distributive property.

#### Goal 3: The learner will solve linear equations and inequalities in one variable.

- 3.1: Solve an equation by using the addition or subtraction property of equality.
- 3.2: Solve an equation by using the multiplication or division property of equality.
- 3.3: Solve an equation by using more than one property of equality.
- **3.4:** Solve an equation graphically.
- 3.5: Solve an equation by using the distributive property.
- 3.6: Solve an equation which contains similar terms.
- 3.7: Find the solution set for a linear inequality when replacement values are given for the variable.
- 3.8: Solve a linear inequality by using transformations.
- **3.9:** Use a linear equation to solve problems.
- **3.10:** Use formulas to solve problems.
- 3.11: Use inequalities to solve problems.

#### Goal 4: The learner will use ratios, proportions, and percents to solve problems.

- 4.1: Write a ratio in lowest terms.
- 4.2: Solve a proportion.
- 4.3: Use a ratio or a proportion to solve problems.
- **4.4:** Solve problems involving percents.

#### Goal 5: The learner will perform operations with real numbers.

- 5.1: Determine the additive or multiplicative inverse of a number.
- 5.2: Distinguish between rational and irrational numbers.
- 5.3: Determine the absolute value of expressions.
- 5.4: Compare real number expressions.
- 5.5: Simplify variable expressions involving addition and subtraction.
- 5.6: Simplify variable expressions involving multiplication and division.
- 5.7: Simplify real number expressions with or without a calculator.
- 5.8: Simplify radicals.
- 5.9: Multiply or divide two radical expressions.
- 5.10: Use a calculator to find the approximate square root of a number.
- 5.11: Add or subtract radical expressions.
- 5.12: Use the Pythagorean Theorem to solve problems.
- Goal 6: The learner will demonstrate an elementary understanding of relations and functions.
  - 6.1: Graph and locate sets of real numbers on a number line.
  - **6.2:** Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.
  - 6.3: Find the distance between two points on a number line.
  - 6.4: Graph a relation on the coordinate plane.
  - 6.5: Graph a relation given an equation and a domain.
  - 6.6: Identify intercepts as numbers.

#### Goal 7: The learner will solve and graph linear equations and inequalities.

- 7.1: Graph an inequality on a number line.
- 7.2: Solve an inequality and graph its solution set.
- 7.3: Determine if data are behaving in a linear fashion.
- 7.4: Find the solution set of open sentences in two variables when given replacement sets for the variables.
- 7.5: Graph a linear equation in two variables.

#### Goal 8: The learner will solve linear equations with two variables.

- 8.1: Solve systems of linear equations.
- 8.2: Use substitution to solve systems of linear equations.
- 8.3: Identify inconsistent systems.
- 8.4: Use a computer or graphics calculator to solve systems of linear equations.

#### Goal 9: The learner will perform operations with polynomials.

- 9.1: Use models to represent polynomials.
- 9.2: Use models to find the sum or difference of two polynomials.
- 9.3: Add and subtract polynomials.
- 9.4: Multiply monomials.
- 9.5: Find an indicated power of a monomial.
- 9.6: Multiply a polynomial by a monomial.
- 9.7: Divide two monomials.

#### Goal 10: The learner will simplify expressions with algebraic fractions.

- **10.1:** Simplify algebraic fractions.
- 10.2: Use the distributive property to simplify fractions.
- **10.3:** Multiply algebraic fractions.
- 10.4: Divide algebraic fractions.

### Subject: Algebra I-B

#### Goal 1: The learner will perform operations with real numbers.

- **1.1:** Add or subtract real numbers.
- 1.2: Multiply or divide real numbers.
- 1.3: Use the order of operations to simplify expressions.
- 1.4: Evaluate exponential expressions.
- 1.5: Evaluate polynomials.
- 1.6: Interpret zero as an exponent.
- 1.7: Interpret negative exponents.
- 1.8: Write numbers in scientific notation and use this notation with the calculator.
- 1.9: Simplify real number expressions with and without a calculator.

## Goal 2: The learner will identify the properties of real numbers and apply them to numerical and algebraic expressions.

- 2.1: Recognize and use properties of addition.
- 2.2: Recognize and use properties of multiplication.
- 2.3: Use the distributive property to simplify expressions.
- 2.4: Use the distributive property to factor a common monomial term in an algebraic expression.
- 2.5: Use number properties to combine like terms.

# Goal 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

- 3.1: Solve an equation by using the addition property of equality and the idea of an additive inverse.
- **3.2:** Solve an equation by using the multiplication property of equality and the idea of a multiplicative inverse.
- 3.3: Solve an equation graphically or by using more than one property of equality.
- **3.4:** Solve an equation which has a variable in both members.
- 3.5: Translate word statements into equations and solve them.
- 3.6: Solve an equation in which the numerical coefficient is a fraction.
- **3.7:** Solve a formula for one of its variables or find the value of a variable when values of other variables are given.
- **3.8:** Use formulas to solve problems.
- **3.9:** Graph an inequality on a number line.
- **3.10:** Find the solution set of a linear inequality when replacement values are given for the variable.
- 3.11: Solve a linear inequality by using transformations.
- 3.12: Use inequalities to solve problems.
- **3.13:** Find the solution set of combined inequalities.
- 3.14: Solve a simple equation involving absolute value.

# Goal 4: The learner will demonstrate an elementary understanding of relations and functions.

- **4.1:** Graph ordered pairs of numbers on the coordinate plane and interpret information related to those sets of points.
- 4.2: Graph a relation on the coordinate plane.
- 4.3: Distinguish between a relation and a function.
- 4.4: Graph a relation given an equation and a domain.
- 4.5: Sketch a reasonable graph for a given relationship.
- 4.6: Interpret a graph in a real-word setting.
- 4.7: Use a computer or graphing calculator to explore the graphs of functions.
- **4.8:** Compare ordered pairs of numbers to the line y = x and interpret the result.

#### Goal 5: The learner will simplify radical expressions which contain variables.

- 5.1: Simplify square root radicals.
- 5.2: Simplify radical expressions involving products and quotients.
- 5.3: Simplify the sums and differences of radical expressions.
- 5.4: Find the approximate square root of a number with and without a calculator.
- 5.5: Compare real number expressions.
- 5.6: Multiply two binomials which contain square roots.
- 5.7: Solve equations which contain radicals.

### Goal 6: The learner will graph and solve systems of linear equations and inequalities.

- 6.1: Solve a system of two linear equations in two variables by graphing.
- 6.2: Solve a system of two linear equations by the substitution method.
- **6.3**: Use the addition or subtraction method to find the solution to a pair of linear equations in two variables.
- 6.4: Use multiplication with the addition or subtraction method to solve a system of linear equations.
- 6.5: Use systems of linear equations to solve problems.

#### Goal 7: The learner will perform operations with polynomials.

- 7.1: Add and subtract polynomials.
- 7.2: Multiply polynomials.
- **7.3:** Divide a polynomial by a monomial.
- 7.4: Divide polynomials.
- 7.5: Find the product of two binomials.
- **7.6:** Factor a trinomial.
- 7.7: Find the square root of a binomial.
- 7.8: Factor a perfect square trinomial.
- **7.9:** Find the product of a sum and a difference.
- 7.10: Factor the difference of two squares.

#### Goal 8: The learner will work with ratios, proportions, and percents.

- 8.1: Simplify ratios involving algebraic expressions.
- 8.2: Solve proportions.
- 8.3: Use ratios and proportions to solve problems.
- 8.4: Solve problems involving percents.

#### Goal 9: The learner will simplify expressions with algebraic fractions.

- 9.1: Find the Least Common Multiple (LCM) of algebraic expressions.
- **9.2:** Add algebraic fractions with like denominators.
- 9.3: Subtract algebraic fractions with like denominators.
- 9.4: Add algebraic fractions with unlike denominators.
- 9.5: Subtract algebraic fractions with unlike denominators.

### Goal 10: The learner will analyze linear equations.

- 10.1: Write linear equations in slope-intercept form.
- 10.2: Find the slope of a non-vertical line given two points on the line.
- **10.3:** Graph a line given its slope and *y*-intercept.
- 10.4: Determine the slope of a line from its graph.
- 10.5: Determine the slope of a line from its equation.

#### Goal 11: The learner will explore, graph, and interpret non-linear equations.

- 11.1: Graph a quadratic equation.
- 11.2: Use an automatic grapher to find the solution to a quadratic equation.
- 11.3: Solve a quadratic equation when one member is in factored form and the other member is zero.
- 11.4: Solve a quadratic equation in which a perfect square equals a constant.
- 11.5: Solve a quadratic equation by factoring.
- 11.6: Understand that the vertex provides the maximum or minimum value of the function.
- 11.7: Solve a quadratic equation by using the quadratic formula.
- **11.8:** Use quadratic equations to solve problems.
- 11.9: Determine if a set of data represents an exponential function.
- 11.10: Use formulas, calculators, and automatic graphers to explore and solve problems involving exponentials.

### Subject: Algebra I

#### Goal 1: The learner will use the language of algebra.

- 1.1: Evaluate algebraic expressions.
- 1.2: Use formulas to solve problems.
- 1.3: Translate word phrases and sentences into expressions and equations and vice versa.
- 1.4: Use the associative, commutative and distributive properties.

#### Goal 2: The learner will perform operations with real numbers.

- 2.1: Simplify real number expressions with and without a calculator.
- **2.2:** Determine the additive or multiplicative inverse of a number.
- 2.3: Determine the absolute value of expressions.
- **2.4:** Raise a real number to an indicated power.
- 2.5: Write numbers in scientific notation and use this notation with a calculator.
- 2.6: Distinguish between rational and irrational numbers.
- 2.7: Find approximations for square roots with and without a calculator.
- 2.8: Simplify radical expressions.
- 2.9: Multiply two binomials which contain square roots.
- 2.10: Compare real number expressions.

#### Goal 3: The learner will solve equations and inequalities with one variable.

- **3.1:** Solve an equation by using the addition property of equality and the idea of additive inverse.
- **3.2:** Solve a simple equation by using the multiplication property of equality and the idea of multiplicative inverse.
- 3.3: Solve an equation graphically and by using more than one property of equality.
- **3.4:** Solve an equation which contains similar terms.
- **3.5:** Solve an equation which has the variable in both members.
- **3.6:** Solve an equation in which the numerical coefficient is a fraction.
- **3.7:** Solve a formula for one of its variables or find the value of a variable when values of the other variables are given.
- **3.8:** Use problem solving skills to solve real-world and "word" problems which involve a linear equation or a formula.
- **3.9:** Solve a simple equation involving absolute value.
- **3.10:** Solve a simple equation containing a radical.
- 3.11: Find the solution set for a linear inequality when replacement values are given for the variable.
- 3.12: Solve a linear inequality by using transformations.
- 3.13: Use inequalities to solve problems.
- **3.14:** Find the solution set of combined inequalities.

# Goal 4: The learner will demonstrate an elementary understanding of relations and functions.

- 4.1: Graph and locate sets of real numbers on a number line.
- **4.2:** Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.
- 4.3: Find the distance between two points on a number line.
- 4.4: Graph the relation on the coordinate plane.
- **4.5:** Distinguish between a relation and a function.
- 4.6: Graph a relation given an equation and domain.
- 4.7: Sketch a reasonable graph for a given relationship.
- 4.8: Interpret a graph in a real-world setting.
- 4.9: Use a computer or graphing calculator to explore the graphs of functions.
- **4.10:** Compare ordered pairs to the line y = x and interpret the results.

#### Goal 5: The learner will graph and use linear equations and inequalities.

- 5.1: Determine if data are behaving in a linear fashion.
- 5.2: Find the solution set of open sentences in two variables when given replacement sets for the variables.
- **5.3:** Graph a linear equation in two variables.
- 5.4: Graph a line given its slope and y-intercept.
- **5.5:** Find the slope of a non-vertical line given the graph of a line or an equation of the line or two points on the line.
- 5.6: Describe the slope in a real-world linear relationship using real-world terms.
- 5.7: Write the slope-intercept form of an equation of a line.
- 5.8: Write the equation of a line given the slope and one point on the line, or two points on the line.
- 5.9: Write the equation of a line which models a set of real data.
- 5.10: Use the line which models real data to make predictions.
- 5.11: Graph a linear inequality in two variables.

#### Goal 6: The learner will graph and solve systems of linear equations and inequalities.

- 6.1: Use a graph to find the solution of a pair of linear equations in two variables.
- 6.2: Graph the solution set of a system of linear inequalities in two variables.
- **6.3:** Use a computer or graphics calculator to solve systems of linear equations.
- 6.4: Use the substitution method to find the solution of a pair of linear equations in two variables.
- **6.5**: Use the addition or subtraction method to find the solution of a pair of linear equations in two variables.
- 6.6: Use multiplication with the addition or subtraction method to solve systems of linear equations.
- 6.7: Use systems of linear equations to solve problems.

#### Goal 7: The learner will perform operations with polynomials.

- 7.1: Add and subtract polynomials.
- 7.2: Multiply monomials.
- 7.3: Find an indicated power of a monomial.
- 7.4: Multiply a polynomial by a monomial.
- 7.5: Find the product of two binomials.
- **7.6:** Multiply two polynomials.
- 7.7: Divide two monomials.
- **7.8:** Divide a polynomial by a monomial.
- 7.9: Find a common monomial factor in a polynomial.
- 7.10: Factor the difference of two squares.
- 7.11: Factor a simple quadratic trinomial.

#### Goal 8: The learner will work with ratios, proportions, and percents.

- 8.1: Simplify ratios involving algebraic expressions.
- 8.2: Solve proportions.
- 8.3: Use ratios and proportions to solve problems.
- 8.4: Solve problems involving percents.

#### Goal 9: The learner will explore, graph, and interpret nonlinear equations.

- 9.1: Graph a quadratic equation.
- 9.2: Use an automatic grapher to find the solution to a quadratic equation.
- 9.3: Solve a quadratic equation when one member is in factored form and the other member is zero.
- 9.4: Solve a second degree equation be factoring.
- **9.5:** Use an automatic grapher to relate the solutions of quadratic equations and the *x*-intercepts.
- 9.6: Understand that the vertex provides the maximum or minimum value of a function.
- 9.7: Solve a quadratic equation in which a perfect square equals a constant.
- 9.8: Solve a quadratic equation by using the quadratic formula.
- 9.9: Use quadratic equations to solve problems.
- 9.10: Determine if a set of data represents an exponential function.
- 9.11: Use formulas, calculators and automatic graphers to explore and solve problems involving exponentials.

### Subject: Technical Math

# Goal 1: The learner will use and read measuring devices and solve problems involving the customary and metric units of measurements.

- 1.1: Find the mass/weight of objects using customary and metric units.
- 1.2: Find the volume/capacity of quantities using customary and metric units.
- **1.3:** Compare and convert measurement units as needed to perform calculations, using a calculator when needed.
- 1.4: Use common measuring tools and solve problems involving the measurements.
- 1.5: Compare measurements to specified tolerances.
- 1.6: Use significant digits to indicate accuracy of measurement.
- 1.7: Select the appropriate measuring tool and units of measure to solve a problem.

#### Goal 2: The learner will use ratios, proportions, and percents to solve problems.

- 2.1: Read, interpret and compare ratios.
- 2.2: Recognize and write proportions from given information.
- 2.3: Distinguish between direct and indirect relationships.
- 2.4: Solve problems involving proportions.
- 2.5: Solve problems involving percents.
- 2.6: Interpret and construct maps and scale drawings.
- 2.7: Find the magnitude and direction of a vector.
- 2.8: Solve problems using signed numbers and vectors.

# Goal 3: The learner will solve geometric problems involving two- and three-dimensional shapes.

- 3.1: Identify polygons and their properties.
- 3.2: Calculate the perimeter, area, and circumference of polygons and circles.
- **3.3:** Identify common three-dimensional shapes.
- 3.4: Calculate the surface area and the volume of three-dimensional shapes.
- 3.5: Solve problems involving common geometric properties.
- **3.6:** Draw auxiliary diagrams to help solve for an unknown dimension or an unknown angle in both plane and solid shapes.
- 3.7: Solve both plane and solid geometry problems that involve a series of successive calculations.

#### Goal 4: The learner will analyze patterns and functions.

- 4.1: Decode and extend patterns.
- **4.2:** Identify the domain and range of mathematical relations and functions.
- 4.3: Distinguish between mathematical relations and functions.
- 4.4: Use patterns and functions to solve problems at home and at work.

#### Goal 5: The learner will solve problems using triangle relationships.

- 5.1: Use the Pythagorean Theorem to find a side of a right triangle.
- **5.2:** Use the characteristics of 3:4:5, 45°-45°-90°, and 30°-60°-90° right triangles to solve practical problems.
- 5.3: Use the ratios for the sine, cosine, and tangent of an angle to solve problems that involve right triangles.
- 5.4: Solve problems involving the Law of Cosines and the Law of Sines.

#### Goal 6: The learner will graph and analyze trigonometric functions.

- 6.1: Draw a graph of sine and cosine waves.
- **6.2:** Find the amplitude, wave length, period, and frequency of sine waves.
- 6.3: Find the phase shift between two sine waves.

#### Goal 7: The learner will use formulas to solve problems.

- 7.1: Translate a problem into an equation.
- 7.2: Evaluate algebraic expressions and formulas when replacement values are given.
- 7.3: Select an appropriate formula to solve a problem.
- 7.4: Solve problems with formulas using a calculator when needed.
- 7.5: Solve problems that involve powers and roots.

#### Goal 8: The learner will solve problems that involve nonlinear equations.

- 8.1: Recognize nonlinear equations (involving squares, square roots and reciprocals of the variable) and become familiar with their graphs.
- 8.2: Solve and graph a nonlinear equation.
- 8.3: Read values from the graph of a nonlinear equation.

#### Goal 9: The learner will solve problems involving systems of equations and inequalities.

- 9.1: Write equations and inequalities in terms of two unknowns.
- 9.2: Solve systems of equations.
- 9.3: Solve problems involving inequalities.
- 9.4: Solve linear programming problems.

#### Goal 10: The learner will use statistics to analyze and solve real-world problems.

- 10.1: Distinguish between mean, mode, and median as measures of central tendency.
- 10.2: Calculate the mean, mode, and median for a set of data.
- **10.3:** Draw and interpret a histogram to represent frequency distribution of data.
- 10.4: Distinguish between range, trend, and standard deviation as measures of variability.
- **10.5:** Interpret the characteristics of a normal curve.
- 10.6: Calculate the range and standard deviation to describe a set of data.

#### Goal 11: The learner will use probability to solve real-world problems.

- **11.1:** Find the probability of some simple events.
- 11.2: Count the number of ways an event can happen.
- 11.3: Draw diagrams and charts to help find the probability of an event.

#### Goal 12: The learner will solve problems using computer technology.

- 12.1: Load and use simple spreadsheet templates to solve practical problems.
- **12.2:** Use an appropriate graphics program to produce bar graphs, circle graphs, and line graphs.
- 12.3: Use an appropriate graphics program to graph linear and nonlinear functions.
- **12.4:** Use an appropriate graphics program to graph one or more curves and determine such characteristics as their slope, roots, and intersection points.
- **12.5:** Use an appropriate graphics program to determine the effect of varying parameters in an equation on the shape of the curve representing the graphed equation.

### Subject: Geometry

- Goal 1: The learner will use concepts of points, lines, and planes in one, two, and three dimensions.
  - 1.1: Identify and name sets of points such as line, ray, segment, and plane.
  - 1.2: Draw representations of points, lines and planes.
  - **1.3:** Identify interior and exterior points in two-dimensional and three-dimensional figures.
  - 1.4: Find the coordinates of a point in a plane or in space.
  - 1.5: Find the length of a segment on a line, in a plane, or in space.
  - **1.6:** Apply the segment addition postulate (definition of betweenness).
  - 1.7: Use lengths to solve problems involving geometric probabilities.
  - **1.8:** Identify the midpoint of a given segment on a line, in a plane or in space.

#### Goal 2: The learner will write a valid proof using a variety of reasoning strategies.

- 2.1: Identify the structure of geometric deductive reasoning (undefined term, postulates, theorems).
- 2.2: State and use properties of equality and inequality.
- 2.3: Write and interpret statements in if-then form.
- 2.4: State the converse, inverse, and contrapositive of the conditional.
- 2.5: Write a proof using a flow diagram.
- 2.6: Write proofs using a two-column format.
- 2.7: Write proofs using a paragraph format.
- 2.8: Write indirect proofs.

### Goal 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

- **3.1:** Use the definitions of adjacent, vertical, linear pair, complementary and supplementary angles to solve problems and write proofs.
- 3.2: Use the angle addition postulate to solve problems and write proofs.
- 3.3: Use the definition of angle bisector to solve problems and write proofs.
- **3.4:** Use the definitions of parallel lines, perpendicular lines, and perpendicular bisectors to solve problems and write proofs.
- **3.5:** Use the relationships which exist between special pairs of angles formed by parallel lines and a transversal to solve problems and write proofs.
- **3.6:** Use slopes to determine if two lines are parallel or perpendicular.
- **3.7:** Write the equation of a line parallel or perpendicular to a given line through a given point.

# Goal 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

- 4.1: Model and describe convex polygons and regular polygons.
- 4.2: Use the measures of the interior and exterior angles of a convex polygon to solve problems.
- 4.3: Use the properties of proportions to solve problems.
- 4.4: Use properties of congruent and similar polygons to solve problems and write proofs.
- 4.5: Investigate a variety of transformations on polygons (tessellation, slide, rotation, flip) in a coordinate plane.
- 4.6: Model and describe regular and non-regular polyhedra.
- 4.7: Identify congruent or similar polyhedra.
- **4.8:** Determine coordinates of the vertices of polygons in a plane.
- 4.9: use coordinate geometry to verify conclusions regarding polygons.

#### Goal 5: The learner will develop and use properties to solve problems and write proofs.

- 5.1: Use the definitions and properties of parallelograms to solve problems and write proofs.
- 5.2: Develop and use properties of rectangles, rhombi, and squares to solve problems and write proofs.
- 5.3: Use the definition and properties of trapezoids and isosceles trapezoids to solve problems and write proofs.
- 5.4: Given quadrilaterals in a coordinate plane, solve problems and construct proofs.

## Goal 6: The learner will develop and use properties of triangles to solve problems and write proofs.

- 6.1: Classify triangles according to sides and angles.
- 6.2: Explore the relationships of the lengths of the sides of a triangle.
- **6.3:** Investigate and apply theorems involving the interior and exterior angles of a triangle.
- 6.4: Use postulates and theorems to prove that two triangles and their corresponding parts are congruent.
- 6.5: Construct congruent triangles using congruency postulates and theorems.
- 6.6: Investigate and apply special theorems involving isosceles triangles.
- 6.7: Investigate and apply definitions and theorems involving altitudes, perpendicular bisectors, and medians.
- 6.8: Investigate and apply the theorem involving the segment joining the midpoints of two sides of a triangle.
- **6.9:** Apply theorems involving segments divided proportionally.
- 6.10: Given triangles in the coordinate plane, solve problems and write proofs.

#### Goal 7: The learner will develop and use properties of right triangles to solve problems.

- 7.1: Find the geometric mean between a pair of numbers.
- 7.2: Use the Pythagorean Theorem and its converse to solve problems.
- 7.3: Use the relationship in a special right triangle to solve problems and write proofs.
- **7.4:** State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.
- 7.5: Use a calculator to apply the definitions of sine, cosine, and tangent to solve problems.
- 7.6: Use coordinate methods to solve problems and write proofs related to right triangles.

### Goal 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

- 8.1: Identify and use the definition of a circle and sets of points related to the circle.
- 8.2: Find the center of a circle which passes through three non-collinear points.
- 8.3: Describe the relationship between tangents and circles.
- 8.4: Apply properties involving arcs and angles of circles.
- 8.5: Apply theorems that relate the chords of a circle, excluding the product theorem.
- 8.6: Apply theorems that relate to the tangents, secants, and radii of a circle, excluding the product theorems.
- 8.7: Describe the relationship between the equation of a circle, its center and radius length.
- 8.8: Discuss the relationships of congruent, similar, and concentric circles.
- 8.9: Explore spheres and sets of points related to the sphere.

# Goal 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

- 9.1: Find the perimeter of a geometric figure.
- 9.2: Find the area of a triangle, parallelogram, trapezoid, and rectangle.
- 9.3: Find the ratio of the perimeters, areas, and volumes of similar geometric figures.
- 9.4: Find the circumference and area of a circle.
- 9.5: Compute arc lengths and the area of sectors of a circle.
- 9.6: Use areas to solve problems involving geometric probability.
- 9.7: Find the lateral area, total area, and volume of a right prism, pyramid, right circular cylinder and cone.
- 9.8: Compute the surface area and volume of a sphere to solve problems.
- 9.9: Use an automatic grapher to solve "Max-Min" problems.

#### Subject: Algebra II

#### Goal 1: The learner will solve, graph and use equations and inequalities.

- 1.1: Solve literal equations and formulas for a specified variable.
- 1.2: Use an automatic grapher to estimate solutions of linear and absolute value equations and inequalities.
- 1.3: Solve and graph equations and inequalities involving absolute value.
- 1.4: Interpret the slope and intercepts of a line.
- **1.5**: Apply the concepts of parallel and perpendicular lines as determined by a comparison of their slopes.
- 1.6: Write and use an equation of a line which models a set of linear data.
- **1.7:** Use an automatic grapher to fit a line to a set of linear data. Interpret the slope, intercepts and quality of fit.

#### Goal 2: The learner will demonstrate an understanding of relations and functions.

- **2.1:** Determine if a given relation is a function.
- 2.2: Identify the domain and range of a relation.
- **2.3:** Use function notation.
- 2.4: Graph relations and functions with and without an automatic grapher.
- 2.5: Find the zeros of a function by examining a graph.
- **2.6:** Solve an inequality by examining the graph of the function.
- 2.7: Communicate graphically, algebraically, and verbally real world phenomena as functions.
- 2.8: Find the composition of functions.
- 2.9: Use iterative definitions of functions.

#### Goal 3: The learner will operate with matrices.

- 3.1: Organize data into an array or a matrix.
- 3.2: Add and subtract matrices with and without calculators.
- 3.3: Use scalar multiplication and multiply matrices with and without calculators.
- 3.4: Solve real-world problems using matrices.
- 3.5: Write and solve systems of linear equations in matrix form.
- **3.6:** Solve matrix equations of the form AX = B using calculators or computers.

#### Goal 4: The learner will graph and solve systems of equations and inequalities.

- 4.1: Solve systems of two equations graphically with and without an automatic grapher.
- 4.2: Solve systems of two equations in two variables.
- 4.3: Solve systems of three equations in three variables.
- 4.4: Use systems of equations and inequalities to solve problems.
- 4.5: Solve systems of inequalities by graphing.
- 4.6: Use linear programming to solve real-world problems.

#### Goal 5: The learner will perform operations and solve problems with polynomials.

- 5.1: Divide one polynomial by another of a lower degree.
- 5.2: Use synthetic division to divide a polynomial by a linear binomial.
- 5.3: Factor polynomials completely.
- 5.4: Use factoring to solve polynomial equations.
- 5.5: Use polynomial equations to solve problems.
- 5.6: Expand powers of binomials using Pascal's triangle of the binomial theorem.
- 5.7: Write a polynomial equation given its solutions.

#### Goal 6: The learner will use rational expressions to solve problems.

- 6.1: Simplify ratios involving algebraic expressions.
- **6.2:** Use expressions involving negative exponents.
- **6.3:** Find sums and differences of rational algebraic expressions.
- 6.4: Find products and quotients of rational algebraic expressions.
- **6.5:** Simplify complex fractions.
- **6.6:** Solve fractional equations.
- 6.7: Solve problems involving fractional equations.
- 6.8: Solve problems of direct and inverse variation.
- 6.9: Use joint and combined variation to solve problems.

#### Goal 7: The learner will solve problems with quadratic equations and inequalities.

- 7.1: Complete the square to solve quadratic equations.
- 7.2: Use the quadratic formula to solve quadratic equations.
- 7.3: Define complex numbers and perform basic operations with them.
- **7.4:** Solve quadratic inequalities.
- 7.5: Determine the solutions of quadratic and other polynomial equations using an automatic grapher.
- 7.6: Solve problems using quadratic equations and inequalities.
- 7.7: Interpret the maximum and minimum values of a quadratic function.
- **7.8:** Use the discriminant of a quadratic equation to determine the nature of the roots and the number of x-intercepts of the graph.
- 7.9: Explore the relationship between coefficients and solutions of a quadratic equation.
- 7.10: Solve equations which contain radical expressions.
- 7.11: Explore complex numbers as solutions to quadratic equations.

#### Goal 8: The learner will use analytic geometry to solve problems.

- 8.1: Use the distance and midpoint formulas.
- **8.2:** Write the equations of and graph circles and parabolas given their geometric properties.
- 8.3: Explore the equations of and graph ellipses and hyperbolas.

Goal 9: The learner will solve problems involving logarithmic and exponential functions.

- **9.1:** Use expressions involving fractional exponents.
- **9.2:** Write an exponential function of the form  $f(x) = a \cdot b^x$  given the base and a point.
- **9.3:** Graph exponential functions of the form  $f(x) = a \cdot b^x$ .
- **9.4:** Use exponential equations of the form  $f(x) = (1+r)^x$ , where r is given as a rate of growth.
- **9.5:** Apply the definitions of logarithms.
- 9.6: Use properties of logarithms and exponents.
- **9.7:** Use logarithms to solve expressions of the form  $a \cdot b^x = c$  for x.

#### Goal 10: The learner will solve problems involving sequences and series.

- 10.1: Generate the terms of an arithmetic series by iteration.
- 10.2: Use a calculator or computer to generate the terms of a geometric series by iteration.
- 10.3: Use summation notation to describe the sums in a series.

#### Goal 11: The learner will solve problems involving probability.

- 11.1: Find the probabilities of independent events by multiplying probabilities.
- **11.2:** Find the probabilities of mutually exclusive events by adding probabilities.
- **11.3:** Use tree diagrams to model situations involving both the multiplication and addition principles.
- 11.4: Understand the meaning of probabilistic statements.

### Subject: Advanced Math

#### Goal 1: The learner will model real-world phenomena using techniques of data analysis.

- 1.1: Recognize a mathematical model of linear, quadratic, exponential, trigonometric, and logarithmic functions.
- **1.2:** Use a scatter plot to determine if a given model is appropriate.
- 1.3: Find the equation of a line that models a bivariate linear data set.
- 1.4: Use the linear least squares method to fit linear data.
- **1.5:** Interpret the slope and *y*-intercept of a linear model.
- **1.6:** Determine the goodness-of-fit of a model using residuals and/or correlation coefficients.
- **1.7:** Use models where appropriate to draw conclusions or make predictions.

# Goal 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

- **2.1:** Sketch graphs of the basic functions (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer).
- **2.2:** Compare information given by local behavior versus global behavior on graphs produced on an automatic grapher.
- **2.3:** Find the domain of a function.
- 2.4: Estimate the range of a function.
- 2.5: Determine the symmetry of a given graph.
- 2.6: Identify continuous and discontinuous functions and locate points of discontinuity.
- 2.7: Graph transformations and combinations of transformations for all basic functions.
- 2.8: Find coordinates of zeros and maximum or minimum points of a given function.
- 2.9: Write the equation of a function from a description of its behavior.
- 2.10: Solve inequalities using an automatic grapher or number line analysis.
- 2.11: Compose two functions and find the domain of the composition.
- 2.12: Analyze a function by decomposing it into simpler functions.
- 2.13: Find the inverse of a function and the domain of the inverse.

#### Goal 3: The learner will graph and use as models polynomial and rational functions.

- 3.1: Find zeros and factors of polynomials algebraically (where appropriate) and/or using an automatic grapher.
- 3.2: Estimate the turning points of polynomial functions using an automatic grapher.
- **3.3:** Find the zeros and vertical asymptotes of a rational function through analysis of the polynomials in the numerator and denominator.
- **3.4:** Find the horizontal asymptote of a rational function by comparing the growth of the polynomials in the numerator and denominator.
- **3.5:** Sketch the graph of a rational function labeling the horizontal and vertical asymptotes and the x- and y-intercepts.

## Goal 4: The learner will graph, transform, and solve problems involving the exponential and logarithmic functions.

- **4.1:** Model growth using transformations of:  $f(x) = ab^x$  and  $f(x) = (1+r)^x$ .
- 4.2: Define and use the logarithmic function as the inverse of the exponential function.
- 4.3: Find the domain of exponential and logarithmic functions.
- **4.4:** Sketch and investigate the graphs of composition of exponential and logarithmic functions with each other and with other basic functions.
- 4.5: Solve equations containing exponential and logarithmic functions using real-world problems.

## Goal 5: The learner will use techniques of data analysis to model nonlinear data from real-world phenomena.

- 5.1: Linearize data using concepts of composition and inverses in order to find a model of data.
- 5.2: Rewrite the linear equation that models linearized data to fit the original curved data.
- 5.3: Discuss the goodness-of-fit of the model used to represent data.
- **5.4:** Use log-log re-expression to linearize data of the form  $y = a \cdot x^b$  by plotting  $(\log x, \log y)$ .

## Goal 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

- 6.1: Use trigonometric functions to model periodic phenomena.
- **6.2:** Express the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine.
- **6.3**: Sketch a graph of each of the six trigonometric functions and identify the period of each.
- 6.4: Recognize and graph transformations of each of the six trigonometric functions.
- 6.5: Use graphs to develop, recognize, and validate trigonometric identities.
- **6.6:** Understand how the values of sine and cosine are represented on the unit circle.
- 6.7: Use the symmetry of the unit circle to develop, recognize, and validate trigonometric identities.
- **6.8**: Understand the relationship between trigonometry in degree mode and trigonometry in radian mode.
- 6.9: Find the radian measure that corresponds to a given angle or arc length.
- 6.10: Solve trigonometric equations and inequalities either algebraically or using an automatic grapher.
- **6.11:** Find the values of inverse trigonometric functions, applying appropriate domain and range restrictions.
- 6.12: Evaluate and graph compositions of trigonometric and inverse trigonometric functions.
- 6.13: Use the Law of Sines and Law of Cosines to solve problems involving triangles and vectors.
- 6.14: Convert complex numbers from rectangular form to polar form.
- 6.15: Convert complex numbers from polar form to rectangular form.
- 6.16: Use DeMoivre's Theorem to find roots and powers of complex numbers.
- 6.17: Use an automatic grapher to explore polar equations.

Goal 7: The learner will use discrete mathematics concepts to solve problems.

- 7.1: Follow an algorithm.
- 7.2: Use operations with matrices and the inverse of a matrix to solve real world problems.
- 7.3: Use permutations and combinations to solve problems.
- 7.4: Classify events as independent or dependent or mutually exclusive.
- 7.5: Interpret data in terms of mean and standard deviation.
- **7.6:** Recognize and use the normal distribution curve to interpret data.
- **7.7:** Find indicated terms in sequences.
- **7.8:** Find the sum of a finite series.
- **7.9:** Find the limit of an infinite sequence.
- 7.10: Determine whether a given series converges or diverges.
- 7.11: Use mathematical induction to establish a generalization.

### Subject: AP Calculus

- Goal 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.
  - 1.1: Define a function and relate the idea of function to real situations.
  - 1.2: Find the domain and range of a function with and without an automatic grapher.
  - 1.3: Find the sum, product, and quotient of two functions.
  - **1.4:** Find the composition of two functions.
  - **1.5:** Find the domain of a composition of two functions.
  - **1.6:** Find and apply the absolute value of a function.
  - **1.7:** Find and use the inverse of a function.
  - 1.8: Determine if a function is odd, even, or neither odd nor even.
  - 1.9: Determine periodicity and amplitude of a function.
  - 1.10: Describe the symmetry of a function.
  - 1.11: Find the asymptotes of a function.
  - **1.12:** Find the zeros of a function.
  - **1.13:** Use  $a^x$  and  $\log_a x$  and their inverse relationships.

#### Goal 2: Find and use limits of various functions.

- 2.1: Find limits of functions by direct substitution.
- 2.2: Find the limit of a quotient of indeterminant form.
- 2.3: Find the limit of a function as the independent variable approaches infinity.
- 2.4: Find the limit of a function by examining its graph.
- 2.5: Recognize the limit of special functions.
- 2.6: Recognize that functions have non-existent limits.

#### Goal 3: Use the definition of continuity.

- **3.1:** Apply the definition of continuity and find excluded values of a discontinuous function.
- **3.2:** Use the statement: "If f is continuous on [a, b], then f has a maximum and a minimum on [a, b]."
- 3.3: Apply the intermediate value theorem.

#### Goal 4: Use the concepts of differential calculus.

- 4.1: Understand the concept of the derivative.
- 4.2: State and apply the definitions of the derivative.
- 4.3: Find the derivatives of elementary functions.
- 4.4: Find the derivative of a sum, product, and quotient.
- 4.5: Find the derivative of a composite function (chain rule).
- 4.6: Find the derivative of an implicitly defined function.
- 4.7: Find the derivative of the inverse of a function.
- **4.8:** Use logarithmic differentiation.
- 4.9: Find derivatives of higher order.
- 4.10: Apply the Mean Value Theorem.
- 4.11: Use the relation between differentiability and continuity.
- 4.12: Use L'Hopital's rule.

#### Goal 5: Apply the concepts of a derivative.

- 5.1: Find the slope of a curve.
- **5.2:** Find the tangent line to a curve.
- 5.3: Find the normal line to a curve.
- 5.4: Use the derivative to approximate values.
- 5.5: Use Newton's Method to approximate the zeros of a function.
- 5.6: Determine where a function is increasing and where it is decreasing.
- 5.7: Find critical points, relative (local) and absolute maximum and minimum points.
- 5.8: Determine the concavity and points of inflection of a function.
- 5.9: Use the graph of the derivative of a function to identify information about the function.
- **5.10:** Solve extreme value problems.
- 5.11: Find the velocity and the acceleration of a particle moving along a line.
- 5.12: Find average rates of change.
- **5.13:** Find instantaneous rates of change.
- **5.14:** Determine related rates of change.

#### Goal 6: Use the concepts of integral calculus.

- 6.1: Find antiderivatives.
- 6.2: Find the distance and velocity from acceleration with initial conditions.
- 6.3: Solve simple (variable separable) first order differential equations.
- **6.4:** Apply solutions of y' = ky to growth and decay problems.
- **6.5:** Use basic integration formulas.
- 6.6: Use substitution to integrate.
- 6.7: Do simple integration by parts.
- 6.8: Approximate the area under a curve using rectangles or trapezoids.
- 6.9: Recognize the definition of the definite integral as the limit of a sum.
- 6.10: Understand the concept of the definite integral.
- **6.11:** Apply properties of the definite integral.
- **6.12:** Use the Fundamental Theorems  $\frac{d}{dx}\int_a^x f(t) dt = f(x)$  and  $\int_a^b f(x) dx = F(b) F(a)$  where F'(x) = f(x).

#### Goal 7: Apply the concept of the definite integral.

- 7.1: Find the average (mean) value of a function on an interval.
- 7.2: Find the area between curves.
- **7.3:** Find the volume of a solid of revolution about the axes or lines parallel to the axes.
- 7.4: Find the volume of a solid with a known cross section.